Mathematical Modeling and Analysis
Los Alamos NATIONAL LABORATORY

Reference Jacobian rezoning strategy for ALE methods on polyhedral grids

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The relationship between the motions of the grid and fluid is an important issue in computational fluid dynamics. There are two choices that are typically made:

1) grid moves along with the fluid (Lagrangian),

2) grid is static (Eulerian).

The major advantage of the Lagrangian approach is a non-diffusive approximation of the advection term; but as the grid flows with the fluid, its elements may become stretched, flattened, or even entangled, causing the simulation to halt.

Arbitrary Lagrangian-Eulerian (ALE) methods were introduced to exploit the advantages of the Lagrangian approach without facing mesh folding. The main idea of the ALE methodology is to move (*rezone*) the computational grid using the fluid flow *only as a guide*.

A good rezoning algorithm should satisfy two competing criteria. First, it should *maintain a good geometrical quality of the computational grid* to minimize the approximation error. Second, it should *keep the rezoned grid adapted to the Lagrangian flow* to better resolve regions of rapid variation of the flow variables.

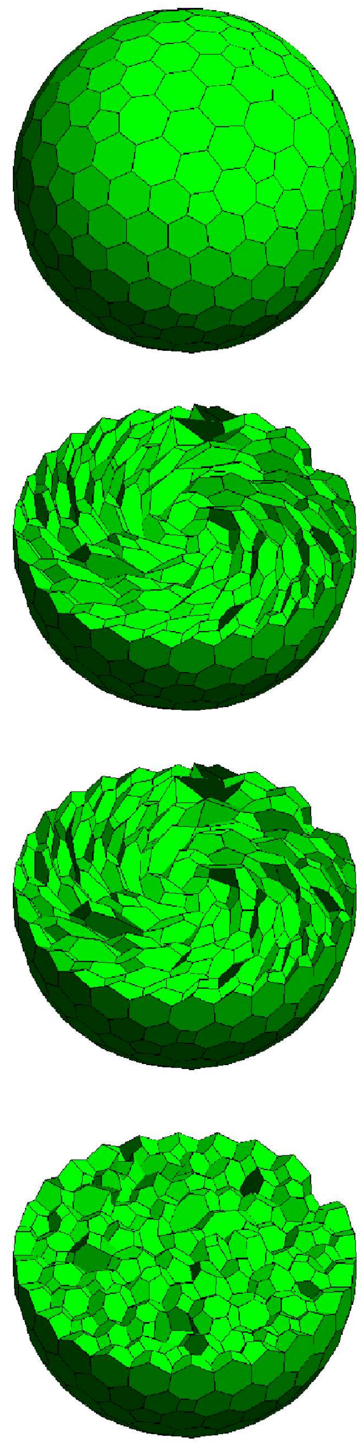
Reference Jacobian (RJ) mesh optimization [1] is positioned as a universal rezoning strategy. It effectively implements a single Jacobi sweep of the global mesh optimization with respect to the given mesh quality measure. By means of numerical experiment we show [2] that RJ rezoning can be successfully used to improve the quality of polyhedral 3D meshes with limited node movement.

the original grid

the original grid (bottom half only)

RJ-optimized grid (bottom half only)

CN-optimized grid (bottom half only)



Resoning (RJ-optimization) vs. complete mesh optimization (CN-optimization). Unlike complete mesh optimization, rezoning effectively eliminates bad elements without violating the long-wave-length distortion of the grid.

The tables below compare RJ rezoning of the grid presented on the figure against complete optimization of the same grid with respect to the Condition Number (CN) quality measure.

Node displacement, **% of the local spacing**		
	RJ-optimized	*CN-optimized*
min	0.1	11.5
avg	7.1	183.3
max	138.4	591.9

Element shape badness			
	Original mesh	*RJ-optimized*	*CN-optimized*
min	1.1	1.2	1.0
avg	12.4	2.1	1.3
max	6662.7	5.9	1.8

	Fraction of elements, %		
Shape badness	*Original mesh*	*RJ-optimized*	*CN-optimized*
1.00 – 1.25	0.9	0.8	26.2
1.25 – 1.50	5.1	4.5	71.0
1.50 – 2.00	36.2	38.2	2.7
2.00 – 3.00	41.0	51.4	0.0
3.00 – 5.00	14.3	5.0	0.0
5.00 – 9.00	2.3	0.2	0.0
9.00 – 17.00	0.0	0.0	0.0
17.00 – ∞	0.2	0.0	0.0

References

[1] P. M. Knupp, L. G. Margolin, and M. Shashkov. Reference Jacobian Optimization-Based Rezone Strategies for Arbitrary Lagrangian-Eulerian Methods. *Journal of Computational Physics*, 176(1):93–128, feb 2002.

[2] V. Dyadechko, R. Garimella, and M. Shashkov. Reference Jacobian rezoning strategy for Arbitrary Lagrangian-Eulerian methods on polyhedral grids. Technical Report LA-UR-05-8159, Los Alamos National Laboratory, Los Alamos, NM, Oct 2005. `http://math.lanl.gov/~vdyadechko/research`

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